A PID Controller with Bounded Torques and Bounded Velocities: Tuning and Experimentation with the PA10-7CE Robot System

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***Abstract*— In this paper we analyze the stability of a saturated PID controller for robot manipulators. Even though this controller has been previously reported in the literature, we now give less restrictive conditions for the control gains in order to ensure local exponential stability of the equilibrium point. The new tuning conditions guarantee that the applied torque signals evolve into prescribed limits, and also the internal velocity commands are not greater than the velocity limits for each actuator. At the end, we show experimental results of the implementation of this controller in a PA10-7CE robot.**

Keywords: PID control, robot control, bounded control.

I. INTRODUCTION

The PID controller is the basic scheme for the position regulation and tracking of industrial robot manipulators. A historical review about this theme can be found in (Santibañez et al., 2010) and (Orrante-Sakanassi et al., 2010). In practice, industrial robots are equipped with a position control computer which produces the desired joint commands for the actuator servo-drivers; these commands are bounded in a similar way than the actuator inputs. In such a sense, very recently Santibanez *et al.*, (2010) proposed a new saturated nonlinear PID regulator for robot manipulators that considers the saturation phenomena of the control computer, the velocity servo–drivers, and also the torque limitations of the actuators.

The contribution in this paper is twofold. First, we recall a variant of the work published in (Santibañez et al., 2010), where the controller, proposed and analyzed by Orrante-Sakanassi & Santibáñez, (2009), is composed by a saturated proportional (P) inner velocity loop, provided by the servo–driver, and a saturated proportional–integral (PI) outer position loop, supplied by the control computer (see Fig. 1); such structure naturally matches the one in practical industrial robots. But now, we present less restrictive conditions for the control gains than those presented in (Orrante-Sakanassi & Santibañez, 2009). Secondly, based on the previous proposed conditions, we show an experimental evaluation of such a nonlinear PID regulator on a PA10-7CE robot arm.

Throughout this paper, we use the notation $\lambda_{\min}\{A(\boldsymbol{x})\}$ and $\lambda_{\max}\{A(\boldsymbol{x})\}$ to indicate the smallest and largest eigenvalues, respectively, of a symmetric positive definite bounded matrix $A(\boldsymbol{x})$, for any $\boldsymbol{x} \in \mathbb{R}^n$. Also, we define $\lambda_{\min}\{A\}$ as the greatest lower bound (infimum) of $\lambda_{\min}\{A(\boldsymbol{x})\}$, for all $\boldsymbol{x} \in \mathbb{R}^n$, that is, $\lambda_{\min}\{A\} = \inf_{\boldsymbol{x}\in\mathbb{R}^n} \lambda_{\min}\{A(\boldsymbol{x})\}$. Similarly, we define $\lambda_{\max}\{A\}$ as the least upper bound (supremum) of $\lambda_{\max}\{A(\boldsymbol{x})\}$, for all $\boldsymbol{x} \in \mathbb{R}^n$, that is, $\lambda_{\max}\{A\} = \sup_{\boldsymbol{x}\in\mathbb{R}^n} \lambda_{\max}\{A(\boldsymbol{x})\}$. The norm of vector $\boldsymbol{x}$ is defined as $\|\boldsymbol{x}\| = \sqrt{\boldsymbol{x}^T\boldsymbol{x}}$ and that of matrix $A(\boldsymbol{x})$ is defined as the corresponding induced norm $\|A(\boldsymbol{x})\| = \sqrt{\lambda_{\max}\{A(\boldsymbol{x})^T A(\boldsymbol{x})\}}$.

II. PRELIMINARIES

A. Robot dynamics

The dynamics of a serial n–link rigid robot, without the effect of friction, can be written as (Spong & Vidyasagar, 1989):

$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} + \boldsymbol{g}(\boldsymbol{q}) = \boldsymbol{\tau} \tag{1}$$

where $\boldsymbol{q} \in \mathbb{R}^n$ is the vector of joint positions, $\dot{\boldsymbol{q}} \in \mathbb{R}^n$ is the vector of joint velocities, $\boldsymbol{\tau} \in \mathbb{R}^n$ is the vector of applied torques, $M(\boldsymbol{q}) \in \mathbb{R}^{n\times n}$ is the symmetric positive definite manipulator inertia matrix, $C(\boldsymbol{q},\dot{\boldsymbol{q}}) \in \mathbb{R}^{n\times n}$ is the matrix of centripetal and Coriolis torques, and $\boldsymbol{g}(\boldsymbol{q}) \in \mathbb{R}^n$ is the vector of gravitational torques, obtained as the gradient of the robot potential energy $\mathcal{U}(\boldsymbol{q})$, i.e.

$$\boldsymbol{g}(\boldsymbol{q}) = \frac{\partial \mathcal{U}(\boldsymbol{q})}{\partial \boldsymbol{q}}. \tag{2}$$

We assume that all the joints of the robot are of revolute type.

B. Properties of the robot dynamics

We recall an important property of the dynamics (1) which is useful in our paper:

Property 1. The gravitational torque vector $\boldsymbol{g}(\boldsymbol{q})$ is bounded for all $\boldsymbol{q} \in \mathbb{R}^n$. This means that there exist finite constants $\gamma_i \geq 0$ such that (Craig, 1998):

$$\sup_{\boldsymbol{q}\in\mathbb{R}^n} |g_i(\boldsymbol{q})| \leq \gamma_i \qquad i = 1,\cdots,n, \tag{3}$$

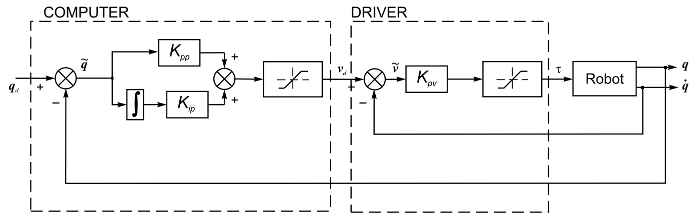

Fig. 1. Scheme of the practical nonlinear PID controller with bounded torques for robot manipulators.

where $g_i(\boldsymbol{q})$ stands for the i-th element of $\boldsymbol{g}(\boldsymbol{q})$. Equivalently, there exists a constant k' such that

$$\|\boldsymbol{g}(\boldsymbol{q})\| \;\leq\; k' \quad \text{for all} \quad \boldsymbol{q} \in \mathbb{R}^n,$$

Furthermore there exists a positive constant k_g such that

$$\left\| \frac{\partial \boldsymbol{g}(\boldsymbol{x})}{\partial \boldsymbol{q}} \right\| \leq k_g,$$

for all $\boldsymbol{q} \in \mathbb{R}^n$, and

$$\|\boldsymbol{g}(\boldsymbol{x}) - \boldsymbol{g}(\boldsymbol{y})\| \leq k_g \|\boldsymbol{x} - \boldsymbol{y}\|,$$

for all $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$.

Moreover, a simple way to compute k_g is:

$$k_g \geq n \left(\max_{i,j,\boldsymbol{q}} \left| \frac{\partial g_i(\boldsymbol{q})}{\partial q_j} \right| \right). \qquad (4)$$

A less restrictive constant k_{g_i} can be computed by:

$$k_{g_i} \geq n \left(\max_{j,\boldsymbol{q}} \left| \frac{\partial g_i(\boldsymbol{q})}{\partial q_j} \right| \right) \qquad (5)$$

where $i = 1, 2, \ldots n$ and $j = 1, 2, \ldots n$. ◇

C. Mean-Value Theorem.

Here, we recall the Mean–Value Theorem, which is a key in finding the less conservative constants k_{g_i} related with the gravitational torque vector.

Theorem 1: (Kelly et al., 2005) Consider the continuous function $f : \mathbb{R}^n \to \mathbb{R}$. If moreover $f(z_1, z_2, \ldots, z_n)$ has continuous partial derivatives then, for any two constant vectors $\boldsymbol{x}, \boldsymbol{y} \in \mathbb{R}^n$, we have

$$f(\boldsymbol{x}) - f(\boldsymbol{y}) = \begin{bmatrix} \left.\frac{\partial f(\boldsymbol{z})}{\partial z_1}\right|_{\boldsymbol{z}=\boldsymbol{\xi}} \\ \left.\frac{\partial f(\boldsymbol{z})}{\partial z_2}\right|_{\boldsymbol{z}=\boldsymbol{\xi}} \\ \vdots \\ \left.\frac{\partial f(\boldsymbol{z})}{\partial z_n}\right|_{\boldsymbol{z}=\boldsymbol{\xi}} \end{bmatrix}^T [\boldsymbol{x} - \boldsymbol{y}] \qquad (6)$$

where $\boldsymbol{\xi} \in \mathbb{R}^n$ is a vector suitably chosen on the line segment which joins vectors $\boldsymbol{x}$ and $\boldsymbol{y}$. ◇

D. Problem formulation

The control scheme presented in this paper involves special saturation functions that fit in the following definition.

Definition 1: (Zavala & Santibañez, 2006) Given some positive constants l and m, with $l < m$, a function $\mathrm{Sat}(x; l, m) : \mathbb{R} \to \mathbb{R} : x \mapsto \mathrm{Sat}(x; l, m)$ is said to be a strictly increasing linear saturation function for (l, m) if it is locally Lipschitz, strictly increasing, C^2 differentiable and satisfies:

1) $\mathrm{Sat}(x; l, m) = x$ when $|x| \leq l$
2) $|\mathrm{Sat}(x; l, m)| < m$ for all $x \in \mathbb{R}$.

◇

For instance, the following saturation function:

$$\mathrm{Sat}(x; l, m) = \begin{cases} -l + (m-l)\tanh\left(\frac{x+l}{m-l}\right) & \text{if } x < -l \\ x & \text{if } |x| \leq l \\ l + (m-l)\tanh\left(\frac{x-l}{m-l}\right) & \text{if } x > l \end{cases} \qquad (7)$$

is a special case of the linear saturations given in Definition 2; n saturation functions can be joined together in an $n \times 1$ saturation function vector denoted by $\mathbf{Sat}(\boldsymbol{x}; \boldsymbol{l}, \boldsymbol{m})$, i.e.,

$$\mathbf{Sat}(\boldsymbol{x}; \boldsymbol{l}, \boldsymbol{m}) = \begin{bmatrix} \mathrm{Sat}(x_1; l_1, m_1) \\ \mathrm{Sat}(x_2; l_2, m_2) \\ \vdots \\ \mathrm{Sat}(x_n; l_n, m_n) \end{bmatrix},$$

where $\boldsymbol{x}, \boldsymbol{l}, \boldsymbol{m} \in \mathbb{R}^n$, that is,

$$\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \; \boldsymbol{l} = \begin{bmatrix} l_1 \\ l_2 \\ \vdots \\ l_n \end{bmatrix}, \; \boldsymbol{m} = \begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_n \end{bmatrix}.$$

Consider the robot dynamics (1), and assume that each joint actuator is able to supply a known maximum torque $\tau_i^{\max}$ and a maximum velocity $v_i^{\max}$, so that:

$$|\tau_i| \leq \tau_i^{\max}, \quad |v_{d_i}| \leq v_i^{\max} \qquad i = 1, \ldots, n \qquad (8)$$

where τ_i and v_{d_i} stand for the i-th entry of vectors τ and v_d, respectively; notice in Fig. 1 that these signals are the outputs of the driver and the control computer.

Assumption 1. The maximum torque $\tau_i^{\max}$ of each actuator satisfies the following condition:

$$\tau_i^{\max} > \gamma_i, \qquad (9)$$

where γ_i was defined in Property 1, for $i = 1, 2, ..., n$. $\diamond$

This assumption means that the robot actuators are able to supply torques in order to hold the robot at rest for all desired joint position $\boldsymbol{q}_d \in \mathbb{R}^n$.

The control problem is to compute the joint torque signals $\boldsymbol{\tau} \in \mathbb{R}^n$, satisfying constraints (8), such that the robot joint positions $\boldsymbol{q}$ tend asymptotically toward the constant desired joint positions $\boldsymbol{q}_d$.

III. Controller Analysis

In this section we present an alternative stability analysis for the nonlinear PID controller originally proposed in (Orrante-Sakanassi & Santibañez, 2009). The controller has the form:

$$\boldsymbol{\tau} = \mathbf{Sat}[K_{pv}[\mathbf{Sat}(K_{pp}\tilde{\boldsymbol{q}} + \boldsymbol{w}^*; \boldsymbol{l}^*_{pi}, \boldsymbol{m}^*_{pi}) - \dot{\boldsymbol{q}}]; \boldsymbol{l}_p, \boldsymbol{m}_p] \tag{10}$$

$$\boldsymbol{w}^* = K_{ip}\int_0^t \tilde{\boldsymbol{q}} \ dr \tag{11}$$

where K_{pv}, K_{pp} and K_{ip} are diagonal positive definite matrices. This control law is formed by two loops (an outer joint position proportional-integral PI loop and an inner joint velocity proportional P loop), and considers the saturation effects existing in the outputs of the control stages (see Fig. 1); $\mathbf{Sat}[K_{pv}[\mathbf{Sat}(K_{pp}\tilde{\boldsymbol{q}} + w; \boldsymbol{l}_{pi}, \boldsymbol{m}_{pi}) - \dot{\boldsymbol{q}}]; \boldsymbol{l}_p, \boldsymbol{m}_p]$ is a vector where each element is a saturation function as in Definition 1 for some $(\boldsymbol{l}_p, \boldsymbol{m}_p)$, where $\boldsymbol{l}_p$ and $\boldsymbol{m}_p$ are vectors whose elements are l_{p_i} and m_{p_i}, respectively, with $i = 1, 2, \ldots, n$. The control law (10)-(11) can be rewritten as:

$$\boldsymbol{\tau} = \mathbf{Sat}[\mathbf{Sat}(K_p\tilde{\boldsymbol{q}} + \boldsymbol{w}; \boldsymbol{l}_{pi}, \boldsymbol{m}_{pi}) - K_v\dot{\boldsymbol{q}}; \boldsymbol{l}_p, \boldsymbol{m}_p] \tag{12}$$

$$\boldsymbol{w} = K_i\int_0^t \tilde{\boldsymbol{q}} \ dr \tag{13}$$

where $K_p = K_{pv}K_{pp}$, $K_i = K_{pv}K_{ip}$, $K_v = K_{pv}$, $\boldsymbol{l}_{pi} = K_{pv}\boldsymbol{l}^*_{pi}$, $\boldsymbol{m}_{pi} = K_{pv}\boldsymbol{m}^*_{pi}$, and fulfills the following assumption.

Assumption 2. The saturation limits of the PI and P loops satisfy:

$$\gamma_i < l_{pi_i} < m_{pi_i} \quad \text{and} \quad \gamma_i < l_{p_i} < m_{p_i} < \tau_i^{\max}. \tag{14}$$

Moreover, the diagonal matrix K_p satisfies

$$\lambda_{\min}\{K_p\} > k_g. \tag{15}$$

$\diamond$

Remark: The saturation constraints of the electronic devices and the actuators are, in fact, hard saturations. However, with the end of carrying out the stability analysis, they can be approximated by linear saturation functions, like those defined in Definition 2, with $l < m$ and l arbitrarily close to m.

In order to simplify the notation, henceforth we will omit, in the argument, the limits of the saturation functions.

A. Closed–loop system

By substituting (12)-(13) into the robot dynamics (1), we obtain

$$\frac{d}{dt}\begin{bmatrix} \tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}} \\ \boldsymbol{w} \end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M(\boldsymbol{q})^{-1}[\mathbf{Sat}[\mathbf{Sat}(K_p\tilde{\boldsymbol{q}} + \boldsymbol{w}) - K_v\dot{\boldsymbol{q}}] \\ -C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q})] \\ K_i\tilde{\boldsymbol{q}} \end{bmatrix} \tag{16}$$

which is an autonomous differential equation with a unique equilibrium point given by $[\tilde{\boldsymbol{q}}^T \ \dot{\boldsymbol{q}}^T \ \boldsymbol{w}^T]^T = [\mathbf{0}^T \ \mathbf{0}^T \ \boldsymbol{g}(\boldsymbol{q}_d)^T]^T \in \mathbb{R}^{3n}$, where we have used Assumption 2 to get that $\mathbf{Sat}(\mathbf{Sat}(\boldsymbol{w})) - \boldsymbol{g}(\boldsymbol{q}_d) = \mathbf{0} \Leftrightarrow \boldsymbol{w} = \boldsymbol{g}(\boldsymbol{q}_d)$. In order to move the equilibrium point of (16) to the origin, we apply the following change of variables $\boldsymbol{x} = \boldsymbol{w} - \boldsymbol{g}(\boldsymbol{q}_d)$.

The closed-loop system (16) can be studied as a singularly perturbed system. To this end, (16) can be rewritten as two first-order differential equations (Orrante-Sakanassi & Santibañez, 2009):

$$\frac{d}{dt}\begin{bmatrix} \tilde{\boldsymbol{q}} \\ \dot{\boldsymbol{q}} \end{bmatrix} = \begin{bmatrix} -\dot{\boldsymbol{q}} \\ M(\boldsymbol{q})^{-1}[\mathbf{Sat}[\mathbf{Sat}(K_p\tilde{\boldsymbol{q}} + \boldsymbol{x} + \boldsymbol{g}(\boldsymbol{q}_d)) \\ -K_v\dot{\boldsymbol{q}}] - C(\boldsymbol{q},\dot{\boldsymbol{q}})\dot{\boldsymbol{q}} - \boldsymbol{g}(\boldsymbol{q})] \end{bmatrix} \tag{17}$$

$$\frac{d}{dt'}\boldsymbol{x} = K_i^*\boldsymbol{h}_1(\boldsymbol{x}) \tag{18}$$

where (17) and (18) are the so-called boundary layer system and reduced system respectively. In the latter $K_i^* = \frac{1}{\varepsilon}K_i$ and $t' = \varepsilon t$ is a new time scale; as $\varepsilon > 0$ is a small parameter then t' is slower than t and $\boldsymbol{x}$ in (17) can be seen as a fixed parameter. Also, $\boldsymbol{h}_1(\boldsymbol{x})$ is the value of $\tilde{\boldsymbol{q}}$ which corresponds to the equilibrium of the boundary layer system (17).

Proposition 1: (Orrante-Sakanassi & Santibañez, 2009) Consider the robot dynamics (1) in closed–loop with the practical saturated PID control law (10)-(11). Under Assumption 2, the equilibrium point of (16) is locally exponentially stable. Besides $|\tau_i(t)| \leq \tau_i^{\max}$ and $|v_{d_i}(t)| \leq v_i^{\max}$ for all $i = 1, 2, ..., n$ and $t \geq 0$. $\diamond$

Proof. See (Orrante-Sakanassi & Santibañez, 2009).

Remark: Note, from Proposition 1, that (15) is a sufficient condition for local exponential stability of the equilibrium point in (16). In the remainder of this section we show how (15) can be replaced by the less restrictive condition

$$k_{p_i} > k_{g_i} \tag{19}$$

obtaining the same results as in Proposition 1.

1) Unique equilibrium of the boundary–layer system: The boundary-layer system (17) corresponds to the robotic system under a Saturated PD Controller with Desired Gravity plus a constant vector $\boldsymbol{x}$, and it has equilibrium points which are the solutions of the nonlinear equations:

$$\begin{aligned} \dot{\boldsymbol{q}} &= \mathbf{0} & (20) \\ \mathbf{Sat}[\mathbf{Sat}[K_p\tilde{\boldsymbol{q}} + \boldsymbol{x} + \boldsymbol{g}(\boldsymbol{q}_d)]] - \boldsymbol{g}(\boldsymbol{q}) &= \mathbf{0}. & (21) \end{aligned}$$

Such equations have a unique solution, provided that Assumption 2 and (19) are satisfied. First, from Assumption 2, we have that (21) can be rewritten as:

$$K_p\tilde{\boldsymbol{q}} + \boldsymbol{x} + \boldsymbol{g}(\boldsymbol{q}_d) - \boldsymbol{g}(\boldsymbol{q}) = \mathbf{0}. \qquad (22)$$

The proof that (22) has a unique solution

$$\tilde{\boldsymbol{q}} = \boldsymbol{h}_1(\boldsymbol{x}) = K_p^{-1}[\boldsymbol{g}(\boldsymbol{q}_d - \boldsymbol{h}_1(\boldsymbol{x})) - \boldsymbol{g}(\boldsymbol{q}_d) - \boldsymbol{x}] \qquad (23)$$

provided that $k_{p_i} > k_{g_i}$; is given in Appendix.

2) Positive definiteness of the Lyapunov function candidate for the boundary-layer system: Consider the Lyapunov function candidate

$$\begin{aligned} W_1(\tilde{\boldsymbol{q}}) = & \frac{1}{2}\dot{\boldsymbol{q}}^T M(\boldsymbol{q})\dot{\boldsymbol{q}} \\ & + \sum_{i=1}^{n} \int_0^{\tilde{q}_i} \mathrm{Sat}[\mathrm{Sat}(K_{p_i} r_i + \boldsymbol{x}_i + g_i(\boldsymbol{q}_d))] dr_i \\ & + \mathcal{U}(\boldsymbol{q}_d - \tilde{\boldsymbol{q}}) \\ & - \sum_{i=1}^{n} \int_0^{h_{1_i}(\boldsymbol{x})} \mathrm{Sat}[\mathrm{Sat}(K_{p_i} r_i + \boldsymbol{x}_i + g_i(\boldsymbol{q}_d))] dr_i \\ & - \mathcal{U}(\boldsymbol{q}_d - \boldsymbol{h}_1(\boldsymbol{x})) \qquad (24) \end{aligned}$$

which is proven to be a positive definite and radially unbounded function provided that (15) is satisfied (Zavala & Santibañez, 2007; Orrante-Sakanassi & Santibañez, 2009). Moreover, following the procedure in (Hernandez-Guzman et al., 2008), it is possible to prove that (24) is a positive definite and radially unbounded function if

$$k_{p_i} > \sum_{j=1}^{n} \max_{q} \left| \frac{\partial g_i(\boldsymbol{q})}{\partial q_j} \right| \qquad (25)$$

Note that (19) implies (25).

3) Negative definiteness of the time derivative of the Lyapunov function for the reduced system: In order to analyze the stability of the reduced system (18) we use the following Lyapunov function

$$V(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T (K_i^*)^{-1}\boldsymbol{x} \qquad (26)$$

whose time derivative is

$$\begin{aligned} \dot{V}(\boldsymbol{x}) &= \boldsymbol{x}^T \boldsymbol{h}_1(\boldsymbol{x}) \\ &= -\boldsymbol{h}_1(\boldsymbol{x})^T[-K_p\boldsymbol{h}_1(\boldsymbol{x}) - \boldsymbol{g}(\boldsymbol{q}_d) + \boldsymbol{g}(\boldsymbol{q}_d - \boldsymbol{h}_1(\boldsymbol{x}))] \end{aligned} \qquad (27)$$

where we have used (23). The negative definiteness of (27), provided that (15) is fulfilled, has been proven in (Orrante-Sakanassi & Santibañez, 2009). The same can be proven if (25) is satisfied (Hernandez-Guzman et al., 2008). We are now ready formulate the following:

Proposition 2: Consider the robot dynamics (1) in closed–loop with the practical saturated PID control law (10). Under (14) and (19), the equilibrium point of (16) is locally exponentially stable. Besides $|\tau_i(t)| \le \tau_i^{\max}$ and $|v_{d_i}(t)| \le v_i^{\max}$ for all $i = 1, 2, ..., n$ and $t \ge 0$.

◇

IV. Experimental results

This section shows the results of a real–time experimental essay on the PA10-7CE robot system. The PA10-7CE robot is a 7–dof redundant manipulator with revolute joints. The numerical values of the parameters for the PA10-7CE are shown in Table I. Table II shows the values of the gains and the saturation limits for each joint of the proposed control scheme (10). It is easy to check that the conditions (14) and (19) are fulfilled. Fig. 2 shows the evolution of the position error for each joint. It can be seen that transient responses are relatively fast (lower than 1 second for joints 4 to 7 and lower than 2 seconds for joints 1 to 3) without overshoot. The steady state error for each joint is lower than 0.4 degrees. Fig. 3 shows the applied torque for each joint. The torques evolve within the prescribed limits. For the joints 4 to 7 the torques sometimes reach the permitted torque limits, thus confirming the stability theoretical result. Fig. 4 shows the velocity references for each joint. The velocity references also evolve within the prescribed limits. For all joints, the velocity references reach, in the first seconds, the velocity limits of the actuators.

TABLE I

Numerical values of the parameters for the PA10-7CE

Parameter	Joint 1	Joint 2	Joint 3	Joint 4	Joint 5	Joint 6	Joint 7	Units
k_{g_i}	0	909.58	216.39	432.25	0.8240	1.3734	0	[N m/rad]
γ_i	0	129.94	30.91	61.75	0.11772	0.1962	0	[N m]
$\tau_i^{\max}$	232	232	100	100	14.5	14.5	14.5	[N m]
$v_i^{\max}$	1	1	2	2	2π	2π	2π	[rad/s]
k_g	909.58							[N m/rad]
k'	147.1513							[N m]

V. Conclusions

In this paper we show that choosing some control parameters with conditions less restrictive than those presented in (Orrante-Sakanassi & Santibañez, 2009) also guarantee local exponential stability of the equilibrium point of (16). It is also guaranteed that, regardless of the initial conditions, the delivered actuator torques and velocities evolve inside the permitted limits. We also have presented results from the real-time practical implementation of this controller on the PA10-7CE robot, by considering the natural saturations of the electronics in the control computer, servo drivers, and actuators.

VI. Acknowledgments

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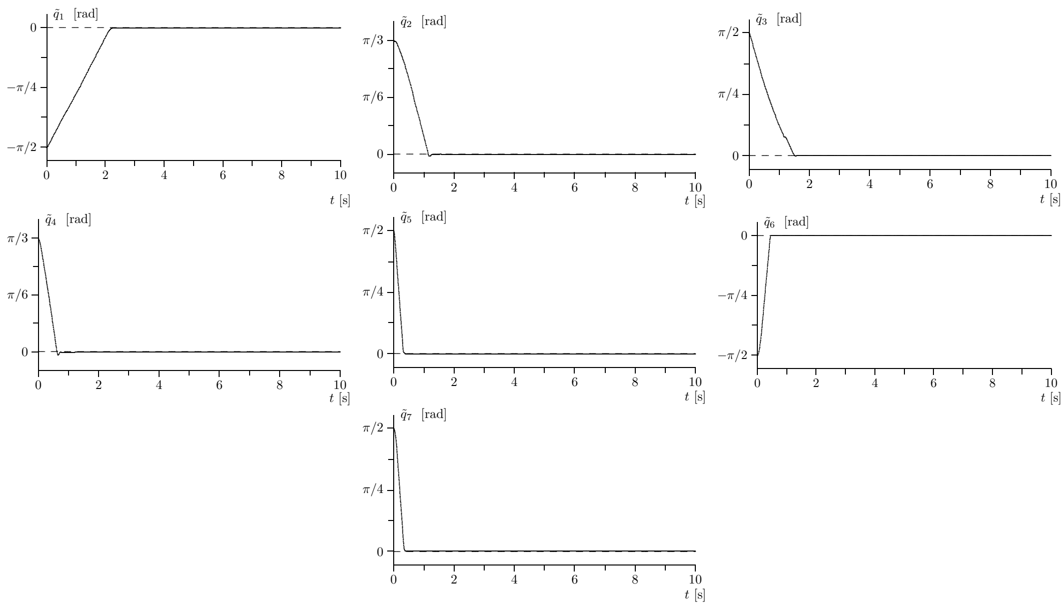

Fig. 2. Position errors

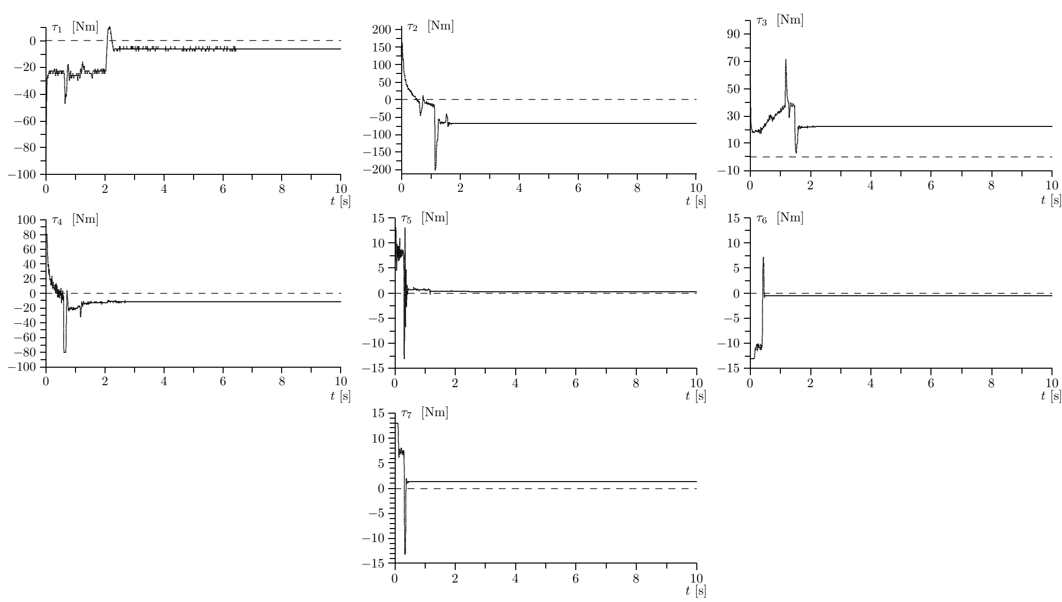

Fig. 3. Applied torques

TABLE II

VALUES OF THE SELECTED CONTROL PARAMETERS

Gain	Joint 1	Joint 2	Joint 3	Joint 4	Joint 5	Joint 6	Joint 7	Units
K_{pp}	10	100	60	60	50	35	30	[1/s]
K_{ip}	0.01	0.01	0.3	0.01	0.5	0.01	0.01	[1/s^2]
K_{pv}	90	150	35	85	10	6	12	[N m s/rad]
l^*_{pi}	0.95	0.95	1.75	1.75	5.5	5.5	5.5	[rad/s]
m^*_{pi}	1	1	1.9	1.9	6	6	6	[rad/s]
l_p	185	185	75	75	12	12	12	[N m]
m_p	200	200	80	80	13	13	13	[N m]

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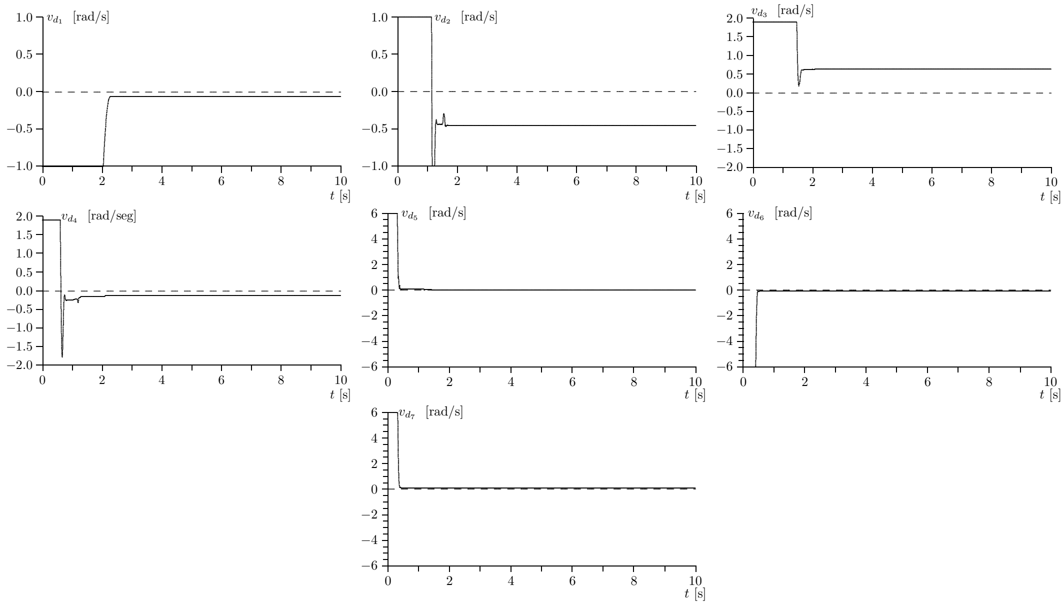

Fig. 4. Velocity errors

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VII. Appendix

In this section we prove that (22) has a unique solution $\tilde{\boldsymbol{q}} = \boldsymbol{h}_1(\boldsymbol{x}) \in \mathbb{R}^n$, provided that:

$$k_{p_i} > k_{g_i} \geq n \left(\max_{\boldsymbol{q},j} \left| \frac{\partial g_i(\boldsymbol{q})}{\partial q_j} \right| \right).$$

To this end, notice that we can rewrite (22) as

$$\tilde{\boldsymbol{q}} = \begin{bmatrix} \tilde{q}_1 \\ \tilde{q}_2 \\ \vdots \\ \tilde{q}_n \end{bmatrix} = \begin{bmatrix} \frac{g_1(\boldsymbol{q}) - g_1(\boldsymbol{q}_d) - x_1}{kp_1} \\ \frac{g_2(\boldsymbol{q}) - g_2(\boldsymbol{q}_d) - x_2}{kp_2} \\ \vdots \\ \frac{g_n(\boldsymbol{q}) - g_n(\boldsymbol{q}_d) - x_n}{kp_n} \end{bmatrix} = \boldsymbol{f}(\tilde{\boldsymbol{q}}, \boldsymbol{q}_d). \quad (28)$$

If $\boldsymbol{f}(\tilde{\boldsymbol{q}}, \boldsymbol{q}_d)$ satisfies the Contraction Mapping Theorem (Kelly et al., 2005; Khalil, 2002), then (28) has a unique solution $\tilde{\boldsymbol{q}}^*$. Having this in mind, we have

$$\|\boldsymbol{f}(\boldsymbol{v}, \boldsymbol{q}_d) - \boldsymbol{f}(\boldsymbol{w}, \boldsymbol{q}_d)\| = \left\| \begin{bmatrix} \frac{g_1(\boldsymbol{q}_d - \boldsymbol{v}) - g_1(\boldsymbol{q}_d - \boldsymbol{w})}{kp_1} \\ \frac{g_2(\boldsymbol{q}_d - \boldsymbol{v}) - g_2(\boldsymbol{q}_d - \boldsymbol{w})}{kp_2} \\ \vdots \\ \frac{g_n(\boldsymbol{q}_d - \boldsymbol{v}) - g_n(\boldsymbol{q}_d - \boldsymbol{w})}{kp_n} \end{bmatrix} \right\| \quad (29)$$

Using Theorem 1, we can rewrite $g_i(\boldsymbol{q}_d - \boldsymbol{v}) - g_i(\boldsymbol{q}_d - \boldsymbol{w})$ as

$$g_i(\boldsymbol{q}_d - \boldsymbol{v}) - g_i(\boldsymbol{q}_d - \boldsymbol{w}) = \left. \frac{\partial g_i(\boldsymbol{z})}{\partial \boldsymbol{z}_1} \right|_{\boldsymbol{z}=\boldsymbol{\xi}_i} (w_1 - v_1)$$
$$+ \left. \frac{\partial g_i(\boldsymbol{z})}{\partial \boldsymbol{z}_2} \right|_{\boldsymbol{z}=\boldsymbol{\xi}_i} (w_2 - v_2) + \ldots + \left. \frac{\partial g_i(\boldsymbol{z})}{\partial \boldsymbol{z}_n} \right|_{\boldsymbol{z}=\boldsymbol{\xi}_i} (w_n - v_n)$$

where $\boldsymbol{\xi}_i$ is a vector on the line segment that joins vectors $\boldsymbol{w}$ and $\boldsymbol{v}$, and, by substituting in (29), we obtain

$$\|\boldsymbol{f}(\boldsymbol{v}, \boldsymbol{q}_d) - \boldsymbol{f}(\boldsymbol{w}, \boldsymbol{q}_d)\| = \|A[\boldsymbol{w} - \boldsymbol{v}]\| \leq \|A\| \|\boldsymbol{w} - \boldsymbol{v}\|$$

where A is the matrix

$$\begin{bmatrix} \frac{1}{kp_1} \frac{\partial g_1(\boldsymbol{z})}{\partial \boldsymbol{z}_1}\Big|_{\boldsymbol{z}=\xi_1} & \frac{1}{kp_1} \frac{\partial g_1(\boldsymbol{z})}{\partial \boldsymbol{z}_2}\Big|_{\boldsymbol{z}=\xi_1} & \cdots & \frac{1}{kp_1} \frac{\partial g_1(\boldsymbol{z})}{\partial \boldsymbol{z}_n}\Big|_{\boldsymbol{z}=\xi_1} \\ \frac{1}{kp_2} \frac{\partial g_2(\boldsymbol{z})}{\partial \boldsymbol{z}_1}\Big|_{\boldsymbol{z}=\xi_2} & \frac{1}{kp_2} \frac{\partial g_2(\boldsymbol{z})}{\partial \boldsymbol{z}_2}\Big|_{\boldsymbol{z}=\xi_2} & \cdots & \frac{1}{kp_2} \frac{\partial g_2(\boldsymbol{z})}{\partial \boldsymbol{z}_n}\Big|_{\boldsymbol{z}=\xi_2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{kp_n} \frac{\partial g_n(\boldsymbol{z})}{\partial \boldsymbol{z}_1}\Big|_{\boldsymbol{z}=\xi_n} & \frac{1}{kp_n} \frac{\partial g_n(\boldsymbol{z})}{\partial \boldsymbol{z}_2}\Big|_{\boldsymbol{z}=\xi_n} & \cdots & \frac{1}{kp_n} \frac{\partial g_n(\boldsymbol{z})}{\partial \boldsymbol{z}_n}\Big|_{\boldsymbol{z}=\xi_n} \end{bmatrix}$$

If $\|A\| < 1$, then $f(\tilde{\boldsymbol{q}}, \boldsymbol{q}_d)$ fulfills the Contraction Mapping Theorem.

Considering (19), it is possible to prove that each element in $A^T A$ fulfills $|A^T A(i,j)| < \frac{1}{n}$. Now, knowing that the eigenvalues from any matrix B, where b_{ij} denotes its ij-th element, fulfill (Horn & Johnson, 1985):

$$|\lambda_k| \leq n \left[\max_{i,j} \{|b_{ij}|\} \right] \quad \forall \quad k = 1, \ldots, n$$

we obtain that

$$\lambda_k\{A^T A\} \leq \lambda_{\max}\{A^T A\} \leq n \left[\max_{i,j} \{|A^T A(i,j)|\} \right] < n \left[\frac{1}{n} \right] = 1$$

and consequently we have that $\|A\| < 1$. Therefore, we get $\|\boldsymbol{f}(\boldsymbol{v}, \boldsymbol{q}_d) - \boldsymbol{f}(\boldsymbol{w}, \boldsymbol{q}_d)\| \leq \|A\| \|\boldsymbol{w} - \boldsymbol{v}\|$ where $\|A\|$ is strictly smaller than the unity. Hence, we have that (22) has a unique solution $\tilde{\boldsymbol{q}} = \boldsymbol{h}_1(\boldsymbol{x}) \in \mathbb{R}^n$ provided that (19) is satisfied.